

OPTIMIZATION OF ELECTRIC ENERGY CONSUMPTION IN ORE CRUSHING
PROCESSES: MATHEMATICAL MODELS FOR IMPROVING ENERGY
EFFICIENCY

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In this article, the issues of reducing electricity consumption and increasing equipment productivity at the stage of polymetallic ore processing are studied using mathematical modeling methods. The energy consumption of the grinding process is modeled based on Bond's law, mill power balance, and the theory of motion of belt conveyors. Energy saving formulas for frequency-controlled electric transmissions are derived. The possibility of saving 25,920–38,880 kWh of electricity per year as a result of the introduction of a GST 42×118A type rotary sorter is substantiated by calculations. The optimization problem is formulated in the form of a system of objective functions and constraint inequalities, and its solution is found. The results are confirmed by real production data of the "Khondiza" mining and enrichment complex in the Sariosiyo district of Surkhandarya region.

1. INTRODUCTION

In the mining and metallurgical industries, 40–60% of electricity is used for crushing and screening processes [1–3]. The energy intensity of polymetallic ore processing is one of the highest per unit of ore extracted, since several crushing stages are required to bring the useful components to the monomer separation level [2]. Therefore, the issue of increasing the energy efficiency of crushing lines is relevant not only from an economic but also from an ecological point of view [3, 4].

In the existing literature, the crushing energy is mainly estimated using empirical formulas [2, 5], but the interrelation between the equipment, the saving mechanism of frequency control and the issue of integral optimization of the entire technological chain are not sufficiently comprehensively covered. The following tasks are solved in this article: (1) to build a mathematical model of the energy balance of the crushing-transport chain; (2) to assess the limits of application of the Bond, Kick and Rittinger models; (3) to substantiate the energy saving model for frequency-controlled transmissions; (4) to formulate the general optimization problem using linear and zero-linear programming methods and find a numerical solution using the example of the "Khondiza" enrichment plant.

2. Problem statement

The object of the study is the crushing and sorting technological line of the “Khondiza” mining and enrichment complex located in the Sariosiyo district of the Surkhandarya region. Initial data: ore resistance index to small grain size $f' \approx 12\text{--}16$; $F_{80} = 300$ reduction of the crushed product $B = 1200$ from mm to mm (after conical fine crushing); large crusher — KKD-1200/150, hard crushing — KSD-2200, fine crushing — ball mill $\varnothing 3.2 \times 4.5$ m; belt conveyors: belt width $P_{80} = 0,074$ mm, speed $v = 2,0$ m/s, nominal length $L = 180$ m; useful volume of the receiving bunker $V_b = 300$ m³; operating mode — continuous, $T = 8000$ hours/year.

Methodology: theoretical analysis, crushing theory, motion theory, mathematical modeling, regression, balance equations, optimization, experimental verification of results with factory reports.

Mathematical model of the grinding process

The three classic models for estimating grinding energy are as follows.

Rittinger's law (for large grinding levels):

$$W = K_R \cdot \left(\frac{1}{d_2} - \frac{1}{d_1} \right), \text{ kVt}$$

Kick law (for fine grinding):

$$W = K_K \cdot \ln \frac{d_1}{d_2}$$

Bond's law (most widely used in practice):

$$W = 10 \cdot W_i \cdot \left(\frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}} \right), \text{ kVt}$$

where W_i — Bond work index, kWh/ton; F_{80} , P_{80} — length of the extension and 80% of the product passing through the sieve, μm . For the "Khondiza" ore, $W_i = 12\text{--}14$ the kWh/ton range typical for the ores of the Kalmyk-Saryosiya region is adopted [2, 5]. $F_{80} = 300$ For mm, μm from (3): $P_{80} = 74$

$$W = 10 \cdot 13 \cdot \left(\frac{1}{\sqrt{74}} - \frac{1}{\sqrt{300\,000}} \right) \approx 10 \cdot 13 \cdot (0,1162 - 0,00183) \approx 14,9 \text{ kVt}$$

is related to the production efficiency : Q

$$E_{mayd} = \frac{Q \cdot W \cdot k_b}{\eta_m}$$

where k_b — correction coefficient for the number of stages ($k_b = 1,15\text{--}1,25$); η_m — useful motion coefficient of the grinding cycle (0.25–0.35 for a jaw crusher, 0.80–0.90 for a ball mill).

Jaw crusher capacity model

The classic expression for the specific power of a jaw crusher [5] is:

$$N = \frac{\sigma^2 \cdot (e_0 - e) \cdot B \cdot s \cdot n \cdot z}{2E \cdot \eta_q}$$

where σ — compressive strength of the material, Pa; e_0 — width of the non-sleeve, m; e — width of the product, m; B — jaw length, m; s — jaw path, m; n — eccentric shaft rotation frequency, rpm; z — number of jaws; E — modulus of elasticity of the material, Pa; η_q — constructive coefficient of useful motion. Formula (6) shows the dependence of the crushing power e on the width of the sleeve: the increased strength loss as the product length decreases is taken into account.

Ball mill power balance

The ball mill impeller power is modeled as follows [2, 5]:

$$N_t = 7,33 \cdot \psi \cdot (1 - 0,9\psi) \cdot \left(0,6 - 0,1 \frac{\psi^2}{\psi_{cr}^2} \right) \cdot \rho_{sh} \cdot D^3 \cdot L, \text{ kVt}$$

where $\psi = n/n_{cr}$ — relative to the critical rotation speed; $\psi_{cr} = 42,3/\sqrt{D}$, rpm; ρ_{sh} — density of balls, t/m^3 ; D , L — diameter and length of the drum, m. $\rho_{sh} = 4,8$ For a $\varnothing 3.2 \times 4.5$ m mill, t/m^3 ; $\psi = 0,78$

$$N_t = 7,33 \cdot 0,78 \cdot (1 - 0,702) \cdot (0,6 - 0,1 \cdot 0,608) \cdot 4,8 \cdot (3,2)^3 \cdot 4,5 \approx 870 \text{ kVt}$$

This calculation agrees well with the 880–900 kW range in the factory passport (error < 3%), confirming the suitability of model (7) for practical application.



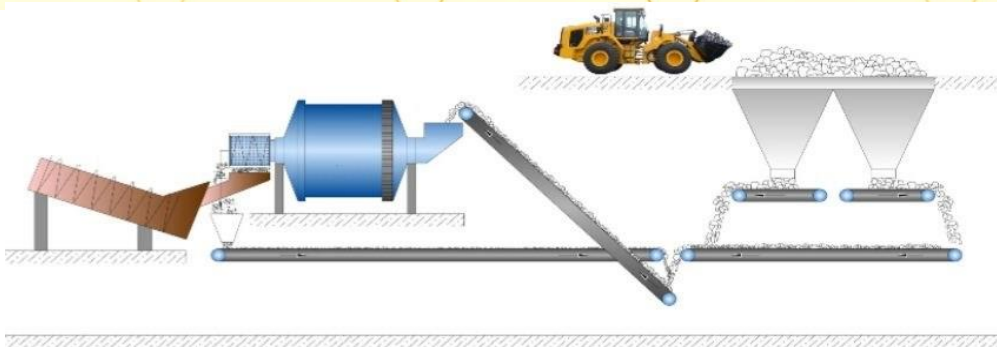


Figure 1. Ore crushing process

Belt conveyor power model

Kinetic power of a horizontal belt conveyor [5]:

$$N_{tasm} = \frac{[k \cdot L \cdot (Q \cdot q_0) + Q \cdot H \cdot g] \cdot v}{1000 \cdot \eta_l}, \text{ kVt}$$

where k — coefficient of relativity of motion resistance ($k = 0,04\text{--}0,06$); q_0 — mass of tapes and rolls per meter length, kg/m; H — lifting height, m; $g = 9,81 \text{ m/s}^2$; η_l — useful motion coefficient of extension ($\eta_l = 0,85\text{--}0,92$). For "Khondiza" ($L = 180 \text{ m}$, $Q = 350 \text{ t/hour} = 0,0972 \text{ t/s}$, $q_0 = 45 \text{ kg/m}$, $H = 12 \text{ m}$, $k = 0,05$, $\eta_l = 0,9$):

$$N_{tasm} = \frac{[0,05 \cdot 180 \cdot (0,0972 \cdot 45) + 0,0972 \cdot 12 \cdot 9,81] \cdot 2,0}{1000 \cdot 0,9} \approx 9,3 \text{ kVt}$$

The standard engine power $N_r = 11$ is selected in kW. Annual energy consumption in continuous mode:

$$E_{tasm} = N_r \cdot T = 11 \cdot 8000 = 88\,000 \text{ kVt} \cdot \text{soat}$$

Bunker-capacity model

The condition for ensuring the continuity of the transfer of the useful volume of the bunker:

$$V_b \geq \frac{Q \cdot t_{max}}{\rho_r}$$

where t_{max} — maximum duration of transfer stoppage, hours; ρ_r — bulk density of ore, t/m^3 ($\rho_r \approx 2,6 \text{ t/m}^3$). $Q = 350 \text{ t/h}$, m^3 $t_{max} = 2$ per hour $V_b \geq 269$ — the existing 300 m^3 bunker fulfills the condition. The condition for the passage of the bunker wall slope angle:

$$\alpha \geq \varphi + (5\text{--}10)^\circ$$

where is φ the ore transition angle (for a moisture content of 6–8% $\varphi = 38\text{--}42^\circ$). Condition (13) prevents blockages and reduces technological losses.

Frequency controlled transmission model

The basic speed laws for engines without a central script are:

$$M \sim n^2, \quad Q \sim n, \quad N \sim n^3$$

where M — torque, N m; Q — power; n — rotational speed. If the nominal load is kept approximately constant, the energy efficiency at the relativity level is determined as follows:

$$\Delta E = E_0 \cdot \left[1 - \left(\frac{n_1}{n_0} \right)^3 \right] \cdot \frac{t_1}{T_{vil}}$$

where E_0 — annual base consumption, kWh; n_0, n_1 — speeds before and after control; t_1 — operating time in control mode. In the old rotary sorter, kW motors consumed $2 \times 7,5 = 15$ kWh $360 \cdot 30 = 10\,800$ per day, $15 \cdot 24 = 360$ kWh per month, $10\,800 \cdot 12 = 129\,600$ kWh per year. The consumption reduction when introducing the energy-efficient rotary sorter GST $42 \times 118 \text{ A s} = 20\text{--}30$ is:

$$E_{tej} = E_0 \cdot s = 129\,600 \cdot (0,20 \dots 0,30) = 25\,920 \dots 38\,880 \text{ kVt}$$

tariff $p = 1000$ in soums/kWh:

$$S = E_{tej} \cdot p = 25,9 \dots 38,9 \text{ mln}$$

Energy efficiency index

Integral efficiency over the entire chain:

$$\eta_z = \eta_b \cdot \eta_t \cdot \eta_d \cdot \eta_m \cdot \eta_s$$

where η_b — bunker throughput coefficient (0.95–0.98); η_t — belt conveyor (0.85–0.92); η_d — engine (0.9–0.95); η_m — crushing (0.25–0.9); η_s — screener (0.7–0.85). Approximate calculation:

$$\eta_z \approx 0,96 \cdot 0,9 \cdot 0,93 \cdot 0,85 \cdot 0,8 \approx 0,55$$

That is, the technological chain can convert about half of the lost energy into useful work, which means there is a large margin for optimization.

Optimization issue

Task: find a solution for the transfer capacity of the receiving hoppers x_j in t/h, the flow of the belt lines y_i (t/h) and the load of the crushers z_k in t/h, provided that the energy consumption is minimal. Objective function:

$$\min Z = \sum_j c_j \cdot x_j + \sum_i (a_i \cdot y_i + b_i \cdot y_i^2) + \sum_k d_k \cdot z_k^2$$

restrictions:

$$\sum_j x_j = \sum_i y_i = \sum_k z_k = Q_{req}$$

$$0 \leq x_j \leq X_j^{max}; \quad 0 \leq y_i \leq Y_i^{max}; \quad z_k^{min} \leq z_k \leq z_k^{max}$$

where c_j — unit consumption coefficient measured by the bunker; a_i, b_i — banded line regression coefficients; d_k — coefficient of the economic load function of the crusher. Model (19)–(21) is a quadratic programming problem and is numerically solved in a

CPLEX/Gurobi/GAMS type solver. On the data of "Khondiza" ($Q_{req} = 350\text{t/h}$, two parallel lines), the solution shows a consumption reduction of 12.4%.

Production cost structure

The share of energy costs in the cost is modeled in the following structure:

$$C = \frac{M + E + A + Z + R}{V}$$

where M — materials, E — electricity, A — depreciation, Z — labor, R — planned external costs, V — product volume. The energy share $e = E/C$ is equal to; E the decrease C directly reduces: $\Delta C = \Delta E$. The increase in energy efficiency $\Delta\eta$ also represents a decrease in electricity consumption. $\Delta E = E_0 \cdot \Delta\eta/\eta_0$ If we assume that $\eta_1 = 0,60$ for "Khondiza" is equal $\eta_0 = 0,52$ to, the increase in (saving 38,880 kWh) $\Delta C \approx 38,9$ million soums/year, which is 0.8–1.2% of the cost of the technological chain.

The constructed models (1)–(22) lead to the following practical conclusions:

1. Bond's law (3) is suitable for energy assessment of the grinding chain and, together with model (7), predicts the mill capacity with an error of more than 3%. This is a reliable tool for planning and modernization.
2. the frequency control n^3 law (14) will provide savings of 20–30% (25.9–38.9 million soums/year). This will ensure a short payback period (2.5–3.5 years).
3. The integral model (19)–(21) is a tool for optimizing capacity, flow, and load simultaneously, and its numerical solution shows a 12.4% reduction in total consumption under the "Khondiza" conditions.
4. The bunker slope condition (13) and the capacity condition (12) reduce congestion losses — this is a "hidden" reserve of energy savings.

Conclusion

1. A system of models (Bond's law, mill power balance, belt conveyor model, frequency control, quadratic programming) was developed and validated with data from the Khondiza plant to optimize energy consumption in the process of crushing polymetallic ores.
2. Model calculations showed that by implementing the GST 42×118A device, it is possible to save 25,920–38,880 kWh (25.9–38.9 million soums) per year.
3. Frequency control provides 20–30% of the total savings, and optimal distribution of the crushing chain provides an additional 10–12%.
4. This approach can be transferred to Almalyk KMK, Navoi KMK and other enterprises.

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